

[Last week](#), we discussed the basics of combinations. Until and unless you have worked a decent bit with combinatorics in high school, the formula of combinations will not be very intuitive. We have already discussed how you can easily think of “selection” in terms of basic counting principle and un-arranging instead of the formula, if you so desire. Today, I would like to discuss some combination questions with constraints. A very common type of such questions asks you to make a committee of  $r$  people out of  $n$  people under some constraints. Let me show you what I mean with the help of some examples.

Question 1: If a committee of 3 people is to be selected from among 6 married couples such that the committee does not include two people who are married to each other, how many such committees are possible?

- (A) 20
- (B) 40
- (C) 80
- (D) 120
- (E) 160

Solution: We have 6 married couples i.e. we have 12 people. Out of these 12 people, we have to choose 3. If there were no constraints on the choice and we could choose any 3 out of these 12, in how many ways could we do that?

12 people and 3 available positions – We use basic counting principle to arrive at  $12 \times 11 \times 10$ . But, recall that we have arranged the 3 people here. To un-arrange, we divide this by  $3!$  to get  $12 \times 11 \times 10 / 3!$  arrangements. This is what we discussed last week.

Moving forward, this question has a constraint – no two people married to each other should be selected i.e. at most one of any two married people could be selected. Say, if we select Mr. X, we cannot select Ms. X and vice versa.

Let's try to use our basic counting principle and un-arranging method here:

You have 12 people and 3 positions. In the first position, you can put any one of the 12 people. In the second position, you can put any one of 10 people (not 11 because the spouse of the person put in 1st position cannot be put in the second position). In the third position you can put any one of 8 people. (We have already selected 2 people for the previous two places and their spouses cannot be selected for the third position so 4 of the 12 people are out.)

Total number of arrangements =  $12 \times 10 \times 8$

But mind you, these are arrangements. We just need a group of 3 people. They don't need to be arranged in the group. So we divide these arrangements by  $3!$  to just 'select' the people.

Number of committees possible =  $12 \times 10 \times 8 / 3! = 160$

I think this was easy, right? Let's look at a little more complex problem now.

Question 2: A group of 10 people consists of 2 married couples and 6 bachelors. A committee of 4 is to be selected from the 10 people. How many different committees can be formed if the committee can consist of at most one married couple?

Solution: We have to select 4 people out of: 6 bachelors and 2 married couples.

The number of ways of selecting any 4 people out of 10 is  $10 \times 9 \times 8 \times 7 / 4! = 210$  (Note here that we are just selecting 4 people. We are not arranging them so we divide by  $4!$ )

The people will get selected in various ways:

1. Four bachelors
2. One from a couple and three bachelors
3. Two from two different couples and two bachelors
4. One couple and two bachelors
5. One couple, one person from a couple, one bachelor
6. Two couples

If we add the number of committees possible in each of these cases, we will get 210. Out of all these cases, only the last one (two couples) has more than one married couple. Instead of calculating the number of different committees that can be formed in each of the first five cases, we can calculate the number of committees in the last case and subtract it from 210.

How many different committees can be formed such that there are 2 couples? Only one since we have only 2 couples. We will have to select both the couples and we will get 4 people.

Number of different committees of 4 people such that there is at most one married couple =  $210 - 1 = 209$ .

Just for practice, let's see how we can calculate the different number of committees that can be formed in each of the first five cases. The sum of all these cases should give us 209.

1. Select 4 bachelors from 6 bachelors in  $6 \cdot 5 \cdot 4 \cdot 3 / 4! = 15$  different committees
2. Select 1 person out of the two couples (4 people) in 4 ways and 3 bachelors from 6 bachelors in  $6 \cdot 5 \cdot 4 / 3! = 20$  ways. So you select the 4 people in  $4 \cdot 20 = 80$  different committees
3. Select 2 people from 2 different couples in  $4 \cdot 2 / 2! = 4$  ways and 2 bachelors from 6 bachelors in  $6 \cdot 5 / 2! = 15$  ways. So you select the 4 people in  $4 \cdot 15 = 60$  different committees
4. Select 1 couple in 2 ways and 2 bachelors from 6 bachelors in  $6 \cdot 5 / 2! = 15$  ways. So you select the 4 people in  $2 \cdot 15 = 30$  different committees
5. Select 1 couple in 2 ways, 1 person from the remaining couple in 2 ways and 1 bachelor from 6 bachelors in 6 ways. So you can select the 4 people in  $2 \cdot 2 \cdot 6 = 24$  different committees

The sum of all these five cases =  $15 + 80 + 60 + 30 + 24 = 209$  different committees (as expected)

Let me add here that combinatorics is a huge topic. We can put up a 100 posts on it and still not exhaust all the content. If there is a particular concept that you would like me to discuss, let me know. Else, I will follow my own train of thought and discuss the most frequently occurring topics.